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ON THE EFFICIENCY OF FERMI ACCELERATION AT RELATIVISTIC SHOCKS

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ABSTRACT

It is shown that Fermi acceleration at an ultra-relativistic shock wave cannot operate on a particle for more than 1 1/2 Fermi cycle (i.e., u $\rightarrow$ d $\rightarrow$ u $\rightarrow$ d) if the particle Larmor radius is much smaller than the coherence length of the magnetic field on both sides of the shock, as is usually assumed. This conclusion is shown to be in excellent agreement with recent numerical simulations. We thus argue that efficient Fermi acceleration at ultra-relativistic shock waves requires significant non-linear processing of the far upstream magnetic field with strong amplification of the small scale magnetic power. The streaming or transverse Weibel instabilities are likely to play a key rôle in this respect.

Subject headings: shock waves – acceleration of particles – cosmic rays

1. INTRODUCTION

Fermi acceleration of charged particles bouncing back and forth a collisionless shock wave is at the heart of a variety of phenomena in high energy astrophysics. According to standard lore, this includes the acceleration of electrons at gamma-ray bursts internal/external relativistic shock waves, whose synchrotron light is interpreted as the prompt/afterglow radiation.

However the inner workings of Fermi acceleration at relativistic shock waves remain the subject of intense study and debate, even in the test particle limit. It has been argued that a universal energy spectral index $s \simeq 2.2 - 2.3$ should be expected (e.g., Bednarz & Ostrowski 1998, Achterberg *et al.* 2001, Lemoine & Pelletier 2003, Ellison & Double 2004, Keshet & Waxman 2005), which would agree nicely with the index that is inferred from gamma-ray bursts observations. Yet recent numerical simulations that include more realistic shock crossing conditions have indicated otherwise (e.g., Lemoine & Revenu 2006, Niemiec & Ostrowski 2006), so that the situation is presently rather confuse. Unfortunately, numerical Monte-Carlo simulations of Fermi acceleration, although they remain a powerful tool, do not shed light on the physical mechanisms at work.

In the present *Letter*, we offer a new analytical discussion of Fermi acceleration in the ultra-relativistic regime. We rely on the observation that, under standard assumptions, the Larmor radius $r_{\rm L}$ of freshly injected particles is much smaller than the coherence length $l_{\rm coh}$ of the upstream magnetic field, and we integrate the equations of motion to first order in the quantity $r_{\rm L}/l_{\rm coh}$ (Section 2). We thus find that a particle cannot execute more than 1 1/2 u $\rightarrow$ d $\rightarrow$ u cycles through the shock before escaping downstream, hence Fermi acceleration is mostly inoperative. This result is related to the very short return timescale in the ultra-relativistic regime: over its trajectory, the particle experiences a nearly coherent and mostly transverse magnetic field, hence the process is akin to superluminal acceleration in regular magnetic fields discussed by Begelman & Kirk (1990). We also show that our analytical predictions agree very well with numerical simulations. Finally, we argue that the (expected) non-linear processing of the magnetic field at ultra-relativistic shock waves and, in particular, the strong amplification of small-scale power, may be the agent of efficient Fermi acceleration (Section 3); we suggest new avenues of research in this direction.

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2. ANALYTICAL TRAJECTORIES

2.1. *Field line curvature*

The upstream magnetic field consists of a regular component B_0 and a turbulent component δB: $\vec B = \vec B_0 + \delta\vec B$, with $\langle B^2\rangle = B_0^2 + \langle\delta B^2\rangle$. The turbulence is defined in the wavenumber range $k_{\rm min} < k < k_{\rm max}$ by its power spectrum $S(k) \propto k^{-\alpha}$, which is normalized according to: $\int {\rm d}^3k\, S(k) = \langle\delta B^2\rangle$. Downstream and upstream magnetic fields are related to each other by the MHD shock jump conditions (see e.g., Kirk & Duffy 1999). In the case of $\gamma-$ray bursts external shocks, the inferred fraction of energy density stored in magnetic turbulence is of the order of a percent (Gruzinov & Waxman 1999) so that the magnetic field can be considered as passive. In this limit, one finds: $B_{\parallel\vert{\rm d}} = B_{\parallel\vert{\rm u}}$, $\vec B_{\perp\vert{\rm d}} = R_{\rm sh}\vec B_{\perp\vert{\rm u}}$; $B_\parallel \equiv \vec B\cdot\vec z$ is the component of the magnetic along the shock normal $\vec z$, $\vec B_\perp$ is the projection of $\vec B$ on the shock front plane $(\vec x,\vec y)$, and subscripts $_{\vert{\rm d}}$ (resp. $_{\vert{\rm u}}$) indicate that the quantity is measured in the downstream (resp. upstream) plasma rest frame. The quantity $R_{\rm sh} \equiv \Gamma_{\rm sh\vert u}\beta_{\rm sh\vert u}/(\Gamma_{\rm sh\vert d}\beta_{\rm sh\vert d})$ is the proper shock compression ratio, expressed in terms of $\beta_{\rm sh\vert u}$ (resp. $\beta_{\rm sh\vert d}$) the shock velocity measured in the upstream (resp. downstream) rest frame and the corresponding Lorentz factor $\Gamma_{\rm sh\vert u}$ (resp. $\Gamma_{\rm sh\vert d}$). In the ultra-relativistic limit ($\Gamma_{\rm sh\vert u} \gg 1$): $R_{\rm sh} \simeq \Gamma_{\rm sh\vert u}\sqrt{8} \gg 1$.

Hence, to an error $\sim \mathcal{O}(1/\Gamma_{\rm sh\vert u})$ on the direction of $\vec B$, it is a good approximation to consider that the magnetic field lies in the transverse $(\vec x,\vec y)$ plane downstream of the shock. The magnetic field at a given point $\vec r_0$ on the shock surface can be written in both downstream and

upstream reference frames as follows:

$$\vec{B}_{|\rm d}(\vec{r}_0) \simeq B_{\perp|\rm d}\cos(\phi_B)\vec{x} + B_{\perp|\rm d}\sin(\phi_B)\vec{y}\ ,$$
$$\vec{B}_{|\rm u}(\vec{r}_0) = B_{\perp|\rm u}\cos(\phi_B)\vec{x} + B_{\perp|\rm u}\sin(\phi_B)\vec{y} + B_{\parallel|\rm u}\vec{z}\ . \quad (1)$$

The phase ϕ_B is invariant under the Lorentz transformation from downstream to upstream; this observation plays a key rôle in the discussion that follows.

We assume for the time being that the Larmor radius $r_{\rm L}$ of the test particle is much smaller than the coherence length of the magnetic field $l_{\rm coh}$; if $\alpha > 3$, $l_{\rm coh} \sim 1/k_{\rm min}$ as the magnetic power is distributed on the largest spatial scales. Then, since the typical u $\to$ d $\to$ u cycle time through the shock is of order $\mathcal{O}\left(r_{\rm L}/\Gamma_{\rm sh|u}\right) \ll l_{\rm coh}$ (Achterberg et al. 2001, Lemoine & Pelletier 2003, Lemoine & Revenu 2006), *in a first approximation one can neglect the magnetic field line curvature over the trajectory of the particle.*

This approximation may be justified as follows. Consider a particle moving over a length scale $l \sim r_{\rm L}/\Gamma_{\rm sh|u} \ll l_{\rm coh}$. The radius of curvature $\mathcal{R}_{>l}$ of the magnetic field on scales larger than l can be calculated as:

$$\mathcal{R}_{>l}^{-1} \equiv \left\langle\left|\frac{(\vec{B}\cdot\vec{\nabla})\vec{B}}{B^2}\right|^2\right\rangle_{>l}^{1/2}\ . \quad (2)$$

Assuming that $\alpha > 3$ (or $l_{\rm coh} \sim 1/k_{\rm min}$) and decomposing $\delta\vec{B}$ in a Fourier series, one finds that indeed, $l/\mathcal{R}_{>l} \sim (\delta B/B)^{-1}\,(l/l_{\rm coh})^{(\alpha-3)/2} \ll 1$. Hence the large scale component is approximately uniform over a length scale l. It is safe to neglect the magnetic power on scales smaller than l since $\delta B_{<l}^2 \sim \delta B^2 (l/l_{\rm coh})^{\alpha-3} \ll \delta B^2$, the latter being comparable to the large scale component. If $\alpha < 3$, similar conclusions apply, since the assumption $r_{\rm L} \ll l_{\rm coh}$ translates into $r_{\rm L} \ll k_{\rm max}^{-1}$, which means that the particle only experiences a smooth large scale magnetic field.

Now, if the magnetic field is approximately regular over the path of the particle in a u $\to$ d $\to$ u cycle, Fermi acceleration in the ultra-relativistic regime becomes similar to superluminal acceleration in a fully regular magnetic field, which is known to be inefficient (Begelman & Kirk 1990). In the following, we extend the discussion of these authors and compare the predictions to numerical simulations of particle propagation in realistic turbulence.

2.2. *Analytical trajectories*

Upstream. The equation of motion reads:

$$\frac{{\rm d}\vec{\beta}}{{\rm d}t} = \Omega_{\rm L}\,\frac{\vec{\beta}\times\vec{B}}{B}\ , \quad (3)$$

with $\vec{\beta}$ the velocity of the particle and $\Omega_{\rm L} = c/r_{\rm L}$ the Larmor frequency. Shock crossing from downstream toward upstream requires $\beta_{\parallel,\,\rm i}^{(\rm u)} \geq \beta_{\rm sh|u}$ with $\beta_{\parallel,\,\rm i}^{(\rm u)}$ the ingress component of the velocity along the shock normal. Hence $\beta_{\perp,\,\rm i}^{(\rm u)} \sim \mathcal{O}(1/\Gamma_{\rm sh|u})$. By working to first order in $1/\Gamma_{\rm sh|u}$, Achterberg *et al.* (2001) were able to obtain analytically the particle trajectory and its direction at shock recrossing u $\to$ d. Assuming that $\phi_B = 0$ so that the transverse component of $\vec{B}$ lies along $\vec{x}$, one obtains the outgoing velocity vector as:

$$\beta_{x,\,\rm f}^{(\rm u)} \simeq \beta_{x,\,\rm i}^{(\rm u)}\ ,$$
$$\beta_{y,\,\rm f}^{(\rm u)} \simeq -\frac{1}{2}\beta_{y,\,\rm i}^{(\rm u)} + \left[\frac{3}{\Gamma_{\rm sh|u}^2} - 3\beta_{x,\,\rm i}^{(\rm u)\,2} - \frac{3}{4}\beta_{y,\,\rm i}^{(\rm u)\,2}\right]^{1/2}\ ,$$
$$\beta_{z,\,\rm f}^{(\rm u)\,2} = 1 - \beta_{x,\,\rm f}^{(\rm u)\,2} - \beta_{y,\,\rm f}^{(\rm u)\,2}\ . \quad (4)$$

Downstream. There we must proceed differently as the return timescale $\sim \mathcal{O}(r_{\rm L}/c)$ can no longer be treated as a small quantity ($r_{\rm L}$ is evaluated in the downstream rest frame; Lemoine & Revenu 2006).

We may assume that $\phi_B = 0$ since the phase is preserved by the Lorentz transformation; furthermore shock compression results in a magnetic field essentially oriented transversally to the shock normal. To this order of approximation, the trajectory along the shock normal reads:

$$\Omega_{\rm L} z(t) = \beta_{\perp,\,\rm i}^{(\rm d)}\sin(\phi_{\rm i})\left[\cos(\Omega_{\rm L}t) - 1\right] + \beta_{\parallel,\,\rm i}^{(\rm d)}\sin(\Omega_{\rm L}t)\ , \quad (5)$$

where $\phi_{\rm i}$ denotes the phase of the ingress velocity vector in the (x, y) plane: $\beta_{x,\,\rm i}^{(\rm d)} \equiv \beta_{\perp,\,\rm i}^{(\rm d)}\cos(\phi_{\rm i})$, $\beta_{y,\,\rm i}^{(\rm d)} \equiv \beta_{\perp,\,\rm i}^{(\rm d)}\sin(\phi_{\rm i})$. The shock front follows the trajectory: $z_{\rm sh|d}(t) = \beta_{\rm sh|d}t$ and return to the shock will occur if and when:

$$\sin(\phi_{\rm i}) = g(\hat{t}) \equiv \frac{\beta_{\rm sh|d}\hat{t} - \beta_{\parallel,\,\rm i}\sin(\hat{t})}{\beta_{\perp,\,\rm i}^{(\rm d)}\left[\cos(\hat{t}) - 1\right]}\ , \quad (6)$$

with $\hat{t} = \Omega_{\rm L}t$. The function $g(\hat{t})$ diverges toward $-\infty$ for $\hat{t} \to 0, 2\pi$ and its derivative is monotonous in the interval $\hat{t} \in]0, 2\pi[$. Hence return to the shock can occur if and only if the maximum of $g(\hat{t})$ exceeds the value $\sin(\phi_{\rm i})$. Note also that $g(\hat{t})$ is always negative since by assumption $\beta_{\parallel,\,\rm i}^{(\rm d)} \leq \beta_{\rm sh|d}$. Therefore *a necessary condition for return to the shock front is $\phi_{\rm i} \in [-\pi, 0]$, or equivalently $\beta_{y,\,\rm i}^{(\rm d)} \leq 0$.*

Once the time $t_{\rm d}$ of shock return has been determined (numerically), the outgoing velocity vector can be derived from the solutions to the equations of motion:

$$\beta_{x,\,\rm f}^{(\rm d)} \simeq \beta_{x,\,\rm i}^{(\rm d)}\ ,$$
$$\beta_{y,\,\rm f}^{(\rm d)} \simeq \beta_{y,\,\rm i}^{(\rm d)}\cos(\Omega_{\rm L}t_{\rm d}) + \beta_{z,\,\rm i}^{(\rm d)}\sin(\Omega_{\rm L}t_{\rm d})$$
$$\beta_{z,\,\rm f}^{(\rm d)} \simeq \beta_{z,\,\rm i}^{(\rm d)}\cos(\Omega_{\rm L}t_{\rm d}) - \beta_{y,\,\rm i}^{(\rm d)}\sin(\Omega_{\rm L}t_{\rm d})\ . \quad (7)$$

2.3. *Mappings: downstream to upstream and vice-versa*

Equations (4) and (7) define mappings from the ingress to the egress angles on either side of the shock. The ingress angles in one rest frame are related to the egress angles in the other rest frame by the Lorentz transformations:

$$\phi_{\rm i}^{(\rm u)} = \phi_{\rm f}^{(\rm d)},\quad \phi_{\rm i}^{(\rm d)} = \phi_{\rm f}^{(\rm u)}\ ,$$
$$\beta_{\parallel,\,\rm i}^{(\rm u)} = \frac{\beta_{\parallel,\,\rm f}^{(\rm d)} + \beta_{\rm rel}}{1 + \beta_{\parallel,\,\rm f}^{(\rm d)}\beta_{\rm rel}},\quad \beta_{\parallel,\,\rm i}^{(\rm d)} = \frac{\beta_{\parallel,\,\rm f}^{(\rm u)} - \beta_{\rm rel}}{1 - \beta_{\parallel,\,\rm f}^{(\rm u)}\beta_{\rm rel}}\ , \quad (8)$$

where $\beta_{\rm rel} \equiv (\beta_{\rm sh|u} - \beta_{\rm sh|d})/(1 - \beta_{\rm sh|u}\beta_{\rm sh|d})$ is the relative velocity between the upstream and downstream rest frames. Using these mappings and transformations, one

can follow the trajectory of a particle. Since the cycle time is of order $r_{\rm L}/\Gamma_{\rm sh} \ll l_{\rm coh}$, it is reasonable to assume that $\phi_B = 0$ remains constant from one Fermi cycle to the next.

Now, in Section 2.2.2, we argued that $\beta_{y,\rm i}^{(\rm d)} \leq 0$ is a necessary condition for the particle to be able to return to the shock. However, as the particle travels upstream and exits back toward downstream, its outgoing velocity is given by Eq. (4) and it can be shown that $\beta_{y,\rm f}^{(\rm u)} \geq 0$ irrespectively of the upstream ingress angle. In effect, for a given $\beta_{y,\rm i}^{(\rm u)}$, the quantity $\beta_{y,\rm f}^{(\rm u)}$ is minimal when $\beta_{x,\rm i}^{(\rm u)}$ is maximal, i.e. when $\beta_{x,\rm i}^{(\rm u)\,2} = 1 - \beta_{\rm sh|u}^2 - \beta_{y,\rm i}^{(\rm u)\,2}$. Then the final $\beta_{y,\rm f}^{(\rm u)} = \left(-\beta_{y,\rm i}^{(\rm u)} + 3|\beta_{y,\rm i}^{(\rm u)}|\right)/2 \geq 0$. The minimum is then 0, which corresponds to a particle entering upstream along $\vec{x}$ (tangentially to the shock surface), i.e. $\beta_{z,\rm i}^{(\rm u)} = \beta_{\rm sh}$, $\beta_{y,\rm i}^{(\rm u)} = 0$. *Hence, if a particle that travels downstream is able once to return to the shock, it will not do so in the subsequent cycle.*

A quantitative assessment of this discussion is shown in Fig. 1 which presents the locii of ingress and egress velocity vectors in the (x, y) plane as seen in the upstream rest frame. The blue area shows the region of egress $\beta_{x,\rm f}^{(\rm u)}$ and $\beta_{y,\rm f}^{(\rm u)}$ (equivalently ingress as seen from downstream) for which the particle is bound to return to the shock. The green circles show the ingress $\beta_{x,\rm i}^{(\rm u)}$ and $\beta_{y,\rm i}^{(\rm u)}$ of a particle that crosses toward upstream. The various circles correspond to different values of $\beta_{z,\rm i}^{(\rm u)}$ upon entry; the radii of these circles are bounded by the shock crossing condition $\beta_{z,\rm i}^{(\rm u)} \geq \beta_{\rm sh|u}$. Finally, the red kidney shaped forms map these ingress upstream velocities into the egress velocities, according to Eq. (4). The fact that these kidney shaped forms do not overlap anywhere with the blue area confirms that at most one and a half cycle $\rm u \to d \to u \to d$ is permitted.

2.4. *Comparison with numerical work*

The previous discussion relies on several approximations, most notably that the field lines can be considered as straight over the trajectory of the particle. Comparison of the previous results with numerical simulations of particle propagation in refined descriptions of the magnetic field are best suited to assess the error that results from these approximations. Figure 2, which shows the contour plot of the return probability defined as a function of $\beta_{x,\rm i}$ and $\beta_{y,\rm i}$, can be directly compared to the blue area of Fig. 1. Indeed, the agreement is excellent. The parameters of the simulations whose results are shown in Fig. 2 are as follows: $r_{\rm L}/L_{\rm max} = 7\times10^{-4}$, $\alpha = 11/3$ (Kolmogorov turbulence), $B_0 = 0$ (pure turbulence) and $\Gamma_{\rm sh|u} = 38$. The numerical procedure used to follow the particle trajectory has been described in Lemoine & Pelletier (2003) and Lemoine & Revenu (2006).

This discussion also explains the results of recent Monte-Carlo simulations. For instance, Niemiec & Ostrowski (2006) report that Fermi acceleration is inefficient in the ultra-relativistic regime for upstream Kolmogorov turbulence; their simulations indicate very steep spectra if any, in good agreement with the present discussion. In contrast, other studies of Fermi acceleration obtain powerlaw spectra of various spectral indices. However, one can check that these latter studies have, one way or another, either assumed an isotropic downstream turbulence, or implicitly marginalized over the angle between the particle trajectory and $\vec{B}_\perp$ at shock crossing, which amounts to picking ϕ_B at random in each half-cycle. In the light of the above discussion, it is then easy

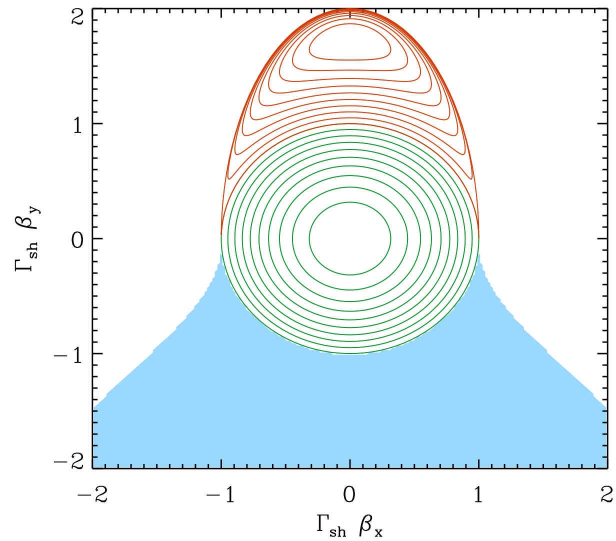

Fig. 1.— Mapping from downstream to upstream and back to upstream as measured in the upstream rest frame, in the plane transverse to the shock front; β_x and β_y are the velocity components in this plane (note the enhancement by $\Gamma_{\rm sh|u}$ on each axis). The solid blue area shows the region of egress upstream coordinates which permits the particle to return to the shock from downstream. The green circled area shows how the original downstream particle population maps upon entering upstream, and the red kidney-shaped region shows the mapping of this population on exit from upstream.

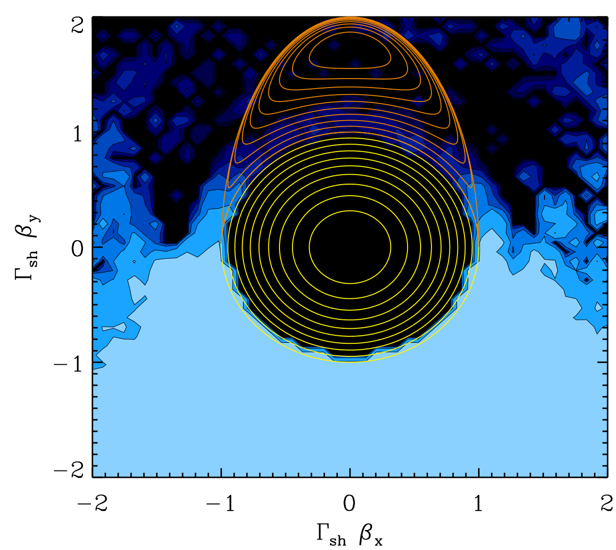

Fig. 2.— Same as Fig. 1 but using results from the numerical simulation of the trajectories of $\sim 10^6$ particles in shock compressed turbulence (see text for details). Each contour represents a drop in return probability by a factor 2.4; the lighter the shading, the higher the return probability, black corresponding to zero. The yellow circles and red kidney-shaped curves are the same as those shown in Fig. 1.

to understand why Fermi acceleration seemed efficient in these studies.

Strictly speaking, our results do not apply to scale invariant turbulence, i.e. $\alpha = 3$. However, results of numerical simulations for this particular case are similar to those shown in Fig. 2; the return probability is non-zero everywhere but it is ten times lower in the kidney-shaped region than in the negative $\beta_{y,\mathrm{i}}$ area. This suggests that quite steep powerlaw spectra should emerge from Fermi acceleration in such turbulence.

3. DISCUSSION

Fermi acceleration is thus inefficient at ultra-relativistic shock waves if the Larmor radius r_{L} of injected particles is much smaller than the coherence length l_{coh} of the turbulent magnetic field on both sides of the shock. This does not mean that Fermi acceleration is bound to fail. In particular, if $r_{\mathrm{L}} \gg l_{\mathrm{coh}}$, powerlaw spectra must emerge as the memory of the magnetic field direction (in the transverse plane) at shock crossing is erased during propagation in small scale turbulence (as we have checked numerically). The final value of the spectral index will depend on the transport properties of the particle in this small-scale turbulence. It is difficult to probe this regime using numerical simulations as integration timescales become large (the scattering timescale $\gg r_{\mathrm{L}}/c$). On analytical grounds, one expects $s \simeq 2.3$ if the small scale turbulence is isotropic downstream of the shock wave (Keshet & Waxman 2005); if it is anisotropic, one ought to expect a different value however.

Although Fermi acceleration could operate efficiently on high energy initial seed particles with $r_{\mathrm{L}} \gg l_{\mathrm{coh}}$, the abundance of such particles is generally so low in realistic astrophysical shock wave environments that the injection efficiency would be extremely small (see Gallant & Achterberg 1999 for instance).

In the above context, the interpretation of the afterglow emission of $\gamma-$ray bursts as the synchrotron radiation of electrons accelerated at the ultra-relativistic external shocks ($\Gamma_{\mathrm{sh|u}} \gtrsim 100$) becomes particularly enlightening. The success of this model indeed requires both efficient Fermi acceleration as well as very significant amplification of the interstellar magnetic field (Gruzinov & Waxman 1999). Hence it is tempting to tie these two facts together and to wonder whether this non-linear MHD processing could not be the agent of efficient Fermi acceleration.

One proposal discussed so far is the transverse Weibel instability which could possibly produce sufficiently strong magnetic fields on very small spatial scales $\sim\ 10^5\,\mathrm{cm}\,(\Gamma_{\mathrm{sh|u}}/10)^{-1/2}(n_e/1\,\mathrm{cm}^{-3})^{-1/2}$ (Medvedev & Loeb 1999); there is however ongoing debate on the lifetime and strength at saturation of the magnetic field (e.g., Wiersma & Achterberg 2004; Lyubarsky & Eichler 2006). Nonetheless, such a small-scale turbulence should result in powerlaw spectra of accelerated particles; albeit the value of the resulting spectral index is not known.

Recently, it has been suggested that the generalization of the streaming instability to the relativistic regime could amplify the magnetic field to the values required by $\gamma-$ray bursts observations (Milosavljevic & Nakar 2006). Due to the very short upstream return timescale $\sim r_{\mathrm{L}}/\Gamma_{\mathrm{sh|u}}$, the particle can never stream too far ahead of the shock so that the turbulence is generated on small scales $\sim\ 10^7 - 10^8\,\mathrm{cm}\ \ll r_{\mathrm{L}}$ (Milosavljevic & Nakar 2006). Therefore, one naturally expects in this case too that Fermi acceleration would be efficient, here as well, one needs to understand the turbulence properties before conclusions can be drawn on the index s.

To summarize, we have shown in Section 2 that Fermi acceleration cannot operate successfully at ultra-relativistic shock waves if one assumes (somewhat naïvely) large-scale turbulence on both sides of the shock wave. The conclusions of the present discussion are thus more optimistic and open a wealth of new possibilities; in particular they suggest that the success of Fermi acceleration is intimately connected with the mechanism of magnetic field amplification in the shock vicinity. The comprehension of Fermi acceleration will eventually require understanding the generation of the magnetic field, deriving the properties of the turbulence as well as characterizing the transport of accelerated particles in this possibly anisotropic turbulence.

Note added: while this work was being completed, a recent preprint by Niemiec *et al.* (2006) appeared, reporting on Fermi acceleration with small-scale turbulence. Although their simulations are limited to $\Gamma_{\mathrm{sh|u}} = 10$, these authors observe that the inclusion of small scale turbulence allows powerlaw spectra to emerge through Fermi acceleration, in good agreement with the above discussion.

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